A rational example of the nuclear fusion reaction at extemely low energy

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All the nuclear phenomena must be understood in the framework of the quamtum theory or better of the quantum field theory. The nuclear fusion at low energy cannot be the exception. In order for the two particles with electric charge Z_1e and Z_2e to fuse, it is necessary to penetrate the repulsive Coulomb potential $V_C(r) = Z_1Z_2e^2/r$ from the region of b =1 Å to the region of a =1 fm.. From the WKB calculation, the transmission coefficient T is

$$T = \exp[-2\tau] \quad with \quad \tau = \int_a^b \frac{\sqrt{2\mu(V(x)-E)}}{\hbar} dx$$

where μ is the reduced mass. In particular, for $Z_1 = Z_2 = 1$ and $\mu = m_N$, in the low energy limit $E \to 0$ τ becomes 120.5 therefore transmission coefficient T becomes forbiddengly small: $T = 2.0 \times 10^{-105}$.

Particle physicists have anticipated for a long time the magnetic monopole. There are various merits to introduce it in addition to restoration of the duality symmetry of the Maxwell equation, which originally has the funny face that the electric objects and the magnetic objects are not treared on the same footing. Since the system of a particle with the magnetic charge ${}^*Q$ and a particle with the electric charge Q has the extra angular momentum $({}^*QQ/c)\hat{r}$ in addition to the orbital angular momentum, if we apply the quantum principle that a component of an angular momentum can assume only the integer multiple of $\hbar/2$, we can obtain Dirac's charge quantization condition:

$${}^*QQ/\hbar c = n/2 \quad with \quad n = 0 \pm 1, \pm 2 \cdots .$$

If we substitute the smallest magnetic charge ${}^*e$ in ${}^*Q$ we obtain $Q = (\hbar c/2{}^*e)n$, which means that the electric charge appearing in Nature is discrete with the pitch $(\hbar c/2{}^*e)$. The magnetic version of the 'fine structure constant' ${}^*e^2/\hbar c$ is obtained by replacing Q and ${}^*Q$ by their smallest value e and ${}^*e$ respectively and by setting $n = 1$:

$${}^*e^2/\hbar c = 1/4e^2/\hbar c = 137/4$$

, which means that the the magnetic Coulomb interaction is super-strong.

Because the magnetic monopole has the very strong magnetic Coulomb field, it can attract half of the nucleus with anomalous magnetic moment such as proton, netron, triton and ${}^3He$ etc., and forms the bound states with these nucleus. The interaction Hamiltonian necessary when we slove the eigenvalue problem is:

$$H_{int} = -\frac{1}{2m_N}\frac{{}^*ee}{\hbar c}(1+\kappa_a)\frac{(\vec{\sigma}\times\hat{r})}{r^2} \quad ,$$

where κ_a is the anomalous magnetic moment measured in nuclear magneton Most of the bound states have the binding energy around 1 MeV. and the radius is around 10^{-12}cm. Once the first nucleus formed the bound state with the monopole, it is rather easy for the second nucleus to approach to the location of the first nucleus-monopole system, because the potential $V(r)$ necessary to penetrate changed from the pure Coulomb to

$$V(r) = -\frac{\kappa_a}{2m_N}\frac{{}^*ee}{\hbar c}\frac{1}{r^2} + \frac{e^2}{r} \quad .$$

This potential has a peak at $r \approx 3 \times 10^{-12}$cm and its height is 17 keV. for the case of the incoming triton, which has large κ_a. When the trapped two nuclei are the proton and triton, or two deuterons, they must quickly fuse to form ${}^4He$, whose transition rate w can be computed by using the standard formula $w = (2\pi/\hbar)| < f|T|i > |^2$, if the wave functions and the nuclear potential are given. Since the spin of ${}^4He$ is zero, it cannot form the bound state with the monopole, therefore the final state is a high speed ${}^4He$ with the produced energy and the fresh monopole. The fresh monopole starts to attract the fuel nuclei with the anomalous magnetic moments anew.

After the brief review of the theory of the magnetic monopole including the derivation of the charge quantization condition, the results and the methods of the calculation of various parameters such as the reaction rate w of the process $p + t \to^4 He$ are given. These parameters must become useful when we design the nuclear fusion reactor in the day in which the magnetic nomopole becomes available.